The importance of missing a defect is $(C_{12})_i = (\Delta I_{md})_i$. The optimal layer thickness corresponds to minimum average risk $SL \to R_{min}$. The results of determining SL_{opt} for one of the samples of organic plastic are presented in Fig. 4. The studies showed that the optimal thickness of the scanning layer in studying moisture transfer in composite materials $SL = 8 \cdot 10^{-3}$ m. The placement error, which for $SL = 8 \cdot 10^{-3}$ m is minimum, has the greatest effect on the result here.

Thus in studying moisture transfer processes in composite materials by the tomographic method some noise level must be taken into account, and the choice of the data processing regime and the thickness of the scanning layer must be approached in a well-founded manner.

NOTATION

$\xi(f)$, distribution of the random variable f; e, base of the natural logarithm; α, shape parameter; Θ, scale parameter; C = 0.5772, Euler's constant; C_{21}, value of a false alarm; C_{12}, importance of missing a defect; $P(H_{21})$, $P(H_{12})$, probability of a false alarm and the probability of missing a defect; Δ, instrumental error; $x_2 - x_1$, range of uncertainty prior to the measurements; $(\Delta y)_i$, increment to the entropy interval owing to the placement error.

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MODELING OF RESERVOIRS IN A BAZHENITE SUITE

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The characteristic features of a structural model of bazhenites, regarded as a collection of horizontal layers, which are formed by a system of overlapping, permeable, lenslike cavities in an impermeable medium, are discussed.

1. Commercial pools of petroleum in the sedimentary deposits of a bazhenite suite are the most promising formations for increasing the reserves of petroleum in the West-Siberian region. The problem of estimating these reserves and developing corresponding computational methods is therefore of great interest. Since bazhenites have a number of unusual properties that distinguish them significantly from other well-known reservoirs, to solve this problem it is first necessary to construct an adequate structural model of bazhenites and the character of the petroleum distribution in them.

Based on modern ideas [1] petroleum-bearing regions in bazhenites are concentrated in permeable, lens-like cavities, oriented parallel to the stratification and occurring in dense, impermeable rocks. In addition, significant, large-scale nonuniformity occurs in both the horizontal and vertical directions. Vertical sections of separate wells reveal permeable intercalations differing in thickness and filtrational characteristics. The sizes and properties of the cavities (lenses) in the same horizontal layer can also vary over a wide range.

These structural features of a reservoir in a bazhenite suite lead to the fact that its global characteristics as a fluid-conducting medium are very unusual. This makes many

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traditional geophysical and especially hydrodynamic methods for calculating petroleum reserves inapplicable. Different hydrodynamic models of bazhenites proposed previously are reviewed and discussed in [2], where it is concluded that the so-called quasiclosed elastoplastic model, in which limited hydraulic coupling between petroleum-bearing cavities is allowed, is most promising for describing the hydraulic phenomena observed in bazhenites. The majority of the most important features of the process of petroleum extraction from bazhenites can indeed all be explained already on the basis of the simplest variant of such a model [3]. There is, however, a fundamental property that is not explained by this formalized model: the hydraulic coupling of the wells in different cases is by no means the same and is often very weak, and pairs of close-lying wells that are definitely not coupled with one another do occur [1, 2]. This indicates that the permeability of the rocks in the matrix separating the petroleum-bearing cavities is very low or generally nonexistent, while the hydraulic coupling occurs primarily as a result of partial overlapping of the cavities or along thin intercalations connecting them.

Permeable petroleum-bearing cavities of a bazhenite reservoir also exhibit a complicated multilevel structure, which, on the one hand, determines the effective mechanical and thermophysical properties of the material of the cavities (an example of the modeling of these properties can be found in [4]) and, on the other, gives rise to deformational processes, which alter the permeability at the bottoms of the wells. This latter property affects the form of the indicator diagrams and the pressure reconstruction curves and on the whole makes it much more difficult to interpret the results of hydraulic tests and to test models at the top hierachical level, i.e., on scales of the order of the size of the cavities. Real deformational processes at the well bottom apparently include the stages of elastic and plastic deformation as well as dilatant fracture and can substantially affect the observed dependences of the output of a well on the bottom-hole pressure, the instantaneous formation pressure, the total yield of petroleum, etc. [5]. This effect was previously studied for a model of a fissured porous material with flat cracks [6].

The situation can also be complicated by pseudoplasticity, which leads to the appearance of a limiting gradient in a uniform medium or, in the presence of exchange between porous blocks and cracks, in a medium with two values of the porosity.

Any attempt to introduce into the model of a bazhenite reservoir the full diversity of structural and other factors, manifested in different scale levels, without complete information is at the present time premature. It is first necessary to construct at least a rough but informative model of a reservoir, taking into account only the large-scale nonuniformity, that could serve as a strong foundation for further analysis as well as to formulate the chief problems in modeling and to indicate the fundamental paths for solving them. Taking this as the goal of this work, in what follows we shall ignore the detailed structure of the material of the cavities as well as the possible change in the properties of the material with time.

2. Based on existing data a bazhenite reservoir can be regarded as a collection of a small number of horizontal layers, which to a first approximation can be regarded as statistically independent and identical. Then one must first study the hypothetical structure of only one such layer, whose local macroscopic properties are random functions of the coordinates.

The determination of the statistical structural characteristics and the effective permeability of each horizon can in principle be studied on the basis of a continuum theory of percolation [7], but the existing factual data are inadequate even for a reliable formulation of the corresponding mathematical problems. We shall thus regard a horizon as a system of cavities in an impermeable surrounding medium. The arrangement of the geometric centers (centers of gravity) of neighboring cavities of different size and shape must obey a definite correlation law, determined by the geological characteristics of the process of formation of bazhenites. Since virtually nothing is known about such correlations, here as a working hypothesis we shall completely neglect their possible existence, making the assumption that the arrangement of the indicated centers is completely independent and arbitrary (and, correspondingly, the cavities freely overlap). Then the distribution of the centers obeys Poisson's law, according to which the probability that N centers occur in an arbitrarily chosen section with area s is given by

TABLE 1. Dependence of the Relative Fraction of the Productive Area of Separate Horizons on the Number of Horizons and the Total Relative Fraction of Unproductive Wells

i	$1-\varphi_*$							
	0,10	0,15	0,20	0,25	0,30	0,35	0,40	0,45
1	0,900	0,850	0,800	0,750	0,700	0,650	0,600	0,550
2	0,684	0,613	0,553	0,500	0,452	0,408	0,368	0,329
3	0,536	0,469	0,415	0,370	0,331	0,295	0,263	0,234
4	0,438	0,378	0,331	0,293	0,260	0,231	0,205	0,181

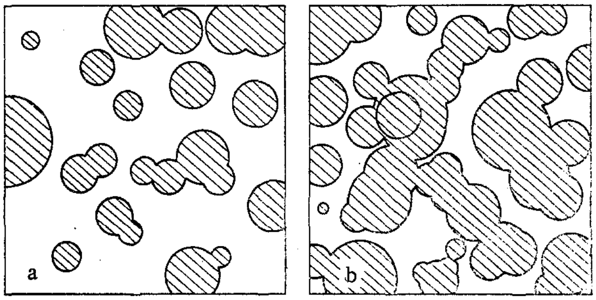

Fig. 1. Structure of the horizon formed by circular lenses with a random uniform distribution of the centers for φ = 0.3 (a) and 0.6 (b); the radii of the lenses obey the distribution (6) with the average value R_0 = 0.13L, where L is the side of the squares; the permeable region is cross-hatched.

$$W_N(s) = \frac{1}{N!}(ns)^N \exp(-ns). \tag{1}$$

Since the probability that one center is present in a small area ds equals nds, it is not difficult to obtain from (1) the probability distribution for the presence of a center of the (j + 1)-st cavity at a distance r from an arbitrarily chosen point r = 0 in the plane. It equals

$$P_{j+1}(r) = \frac{2\pi n}{j!}(\pi r^2 n)^j \exp(-\pi r^2 n)\, r. \tag{2}$$

From here we determine the average distance between the centers of the nearest cavities and the standard deviation of the true distance from the average distance (variance of the distribution):

$$\begin{gathered} \langle r \rangle = \int_0^\infty r P_1(r)\, dr = \frac{1}{2\sqrt{n}}, \\ \langle (r - \langle r \rangle)^2 \rangle^{1/2} = \left(\int_0^\infty \left(r - \frac{1}{2\sqrt{n}} \right)^2 P_1(r)\, dr \right)^{1/2} = \sqrt{\frac{4-\pi}{4\pi n}}. \end{gathered} \tag{3}$$

The probability that some arbitrarily chosen point lies outside all f(s)ds cavities with areas in the interval (s, s + ds) equals, according to (1), exp(-ns). Using the theorem for multiplication of probabilities, we obtain the following formula for the probability of the event that this point lies in the matrix, i.e., not in any of the cavities whose areas are distributed with density f(s),

$$1 - \varphi = \exp\left(-n \int_0^\infty s f(s)\, ds\right), \tag{4}$$

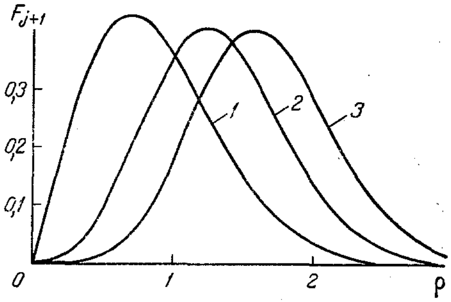

Fig. 2. Reduced distributions of the nearest neighbors $F_{j+1} = (2\sqrt{\pi n})^{-1}P_{j+1}$ as a function of the dimensional distance $\rho = \sqrt{\pi n}r$; the numbers on the curves are the values of j + 1.

which automatically takes into account the possibility that the cavities overlap and is a simple generalization of the formula following from the Mampel's theory of the formation and growth of circular nuclei of a new phase on a flat surface [8]. Obviously, φ is the relative fraction of the area of a given horizon occupied by permeable cavities. If there are i horizons, then because they are assumed to be identical and independent we obtain the following formula for the probability that a well intersects all horizons without hitting a cavity in any of them

$$1-\varphi_{*}=\exp\left(-ni\int_{0}^{\infty}sf(s)\,ds\right). \tag{5}$$

The quantity $1-\varphi_{*}$ can in principle be determined from blocks of data over many wells as the relative fraction of nonproductive wells, and if the number of horizons is known, it is easy to find from here the quantity φ, characterizing each of them. The values of φ, corresponding to different values of $1-\varphi_{*}$ and i, are given in Table 1; this quantity is, of course, of primary importance for evaluating petroleum reserves.

To further describe the topological characteristics of the horizons under study it is necessary to specify the distribution of cavities over the parameters characterizing their shape, orientation in the plane, and size. There is virtually no information about such distributions, so that we shall simply model the cavity by circular lenses, either having the same radius R_0 or radii distributed with density

$$f(R)=\frac{\pi}{2R_0^2}R\exp\left(-\frac{\pi R^2}{4R_0^2}\right),\quad R_0\equiv\langle R\rangle \tag{6}$$

(this function corresponds to a Gaussian distribution of the lengths of segments connecting some point in the permeable region with the closest boundaries of the lens and laid along two perpendicular axes intersecting at this point).

From (4) and (5) we obtain simply for lenses of the same size

$$1-\varphi_{*}=(1-\varphi)^{i}=\exp(-\pi R_0^2 ni),$$

while for lenses distributed according to the law (6) we have

$$1-\varphi_{*}=(1-\varphi)^{i}=\exp(-4R_0^2 ni).$$

The topology of a horizon with lenses with the size distribution (6) is illustrated in Fig. 1 for two values of the relative fraction of the lenses φ. The structures shown are a separate realization of the pattern of disks thrown randomly on a rectangular area with the radius of each disk chosen randomly in accordance with (6).

Using (2) we obtain a formula for the probability of the event that some lens with radius R does not overlap any lens with radius R_1:

$$P|_{r>R+R_1} = \int\limits_{R+R_1}^{\infty} P_1(r)\,dr = \exp[-\pi(R+R_1)^2 n]. \tag{7}$$

The probability that the indicated lens does not overlap any of the other lenses obeying the distribution (6)

$$Q_1(R) = \int\limits_0^{\infty} P|_{r>R+R_1} f(R_1)\,dR_1 = \frac{1}{4R_0^2}\left(n + \frac{1}{4R_0^2}\right)^{-1} \times$$
$$\times [1 - \sqrt{\pi}\,\gamma \exp(\gamma^2)\operatorname{erfc}(\gamma)] \exp(-\pi R^2 n),\ \ \gamma = \sqrt{\pi}\, Rn\left(n + \frac{1}{4R_0^2}\right)^{-1/2}, \tag{8}$$

determines the probability of encountering in each horizon clusters consisting of only one lens with a given radius. The overall relative fraction of isolated lenses can be obtained by averaging (8) with the help of (6).

For a system of identical lenses we obtain from (7) instead of (8)

$$Q_1 = \exp(-4\pi R_0^2). \tag{9}$$

The probability that the distance between the closest centers of lenses with radii R and R_1 exceeds $|R - R_1|$ is given by

$$P|_{r>|R-R_1|} = \exp[-\pi(R-R_1)^2 n]. \tag{10}$$

Subtracting this quantity from unity and integrating the result using the weight (6) with $R = R_1$ in the interval (R, ∞) gives the relative fraction of "phantom lenses" with radius R, contained wholly within lenses with a larger radius. One such phantom lens is shown in Fig. 1.

Finally the formula (2) determines the probability that the centers of the first, second, etc. neighboring lenses located closest to some point lie at a certain distance from this point. These probabilities are useful for choosing locations for new wells in the vicinity of unproductive wells. They all have maxima for definite values of r; for example, the maximum of the probability distribution of a lens center closest to the point is reached for $r = 1/\sqrt{2\pi n}$, which is somewhat less than the average distance between neighboring centers from (3). Because of their practical importance the functions $P_{j+1}(r)$ for small j are shown in Fig. 2.

The relative fraction of lenses with a given radius appearing among clusters of only two lenses, the average concentration of such clusters, as well as their average area can be determined in an analogous manner. The calculations can also in principle be extended to clusters consisting of a large number of lenses, but the purely computational difficulties grow very rapidly as the number of lenses in the clusters increases. If Q_m is the probability of finding a lens in a cluster of m coupled lenses (with m = 1 this probability is obtained by averaging (8) over the distribution (6), while for a system of identical lenses it is determined by (9)) and s_m is the average area of such an m-cluster, then the average number of m-clusters per unit area equals $(n/m)Q_m$, while the area occupied by them is $(n/m)Q_m s_m$. Obviously

$$n \sum_{m=1}^{\infty} \frac{1}{m} Q_m s_m = \varphi, \tag{11}$$

where φ is defined in (4). Correspondingly the quantities $(n/\varphi m)Q_m s_m$ determine the probability that a productive well occurs precisely in an m-cluster of the horizon under study, while s_m is the average area that is in principle drained by such a well in the case that the gaps between the lenses are impermeable. In this case two productive wells are hydraulically coupled with one another if they belong to the same cluster, and they are completely uncoupled if belong to different clusters. If the matrix has a finite though low permeability, then weak hydraulic coupling is also possible between wells draining different

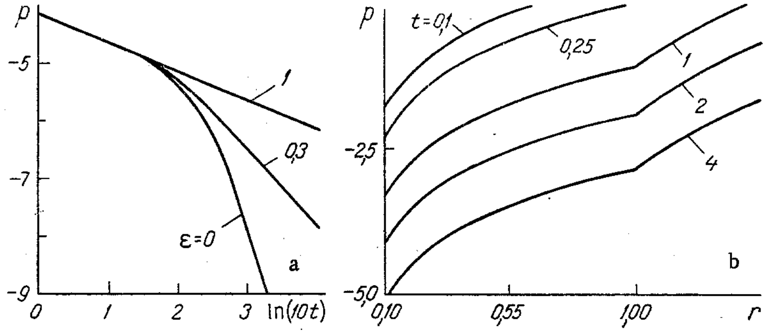

Fig. 3. Dependence of the pressure at the well bottom (relative units) on the dimensionless time t for different values of ε (a) and the mean distribution of the pressure inside (r < 1) and outside the lens for different values of t (b) for a well with the dimensionless radius 0.01 in a regime with a constant output.

clusters. Aside from the average values of Q_m and s_m, for small m it is also possible to find, with the help of direct methods analogous to those employed above, the characteristics of the corresponding distributions.

3. If φ is much lower than the percolation threshold, corresponding to the formation of an infinite cluster consisting of an infinite number of lenses (and equal to 0.5 in the case under study [9]), then in the sum (11) only the first several terms, corresponding to clusters consisting of a small number of lenses, are important; in this case the required calculations can be performed directly, as indicated above.

The situation is completely different near the percolation threshold, when clusters consisting of a large number of overlapping lense become important; in this case much depends on whether or not the real relative fraction of the productive surface of the horizon exceeds the value 0.5, corresponding to the percolation threshold [7, 9, 10]. Unfortunately at the present time there are no complete data that would permit answering this question equivocally. If the relative fraction of unproductive wells equals 10-30%, then as follows from Table 1, depending on the number of independent horizons, both situations can occur. In any case, however, there is no doubt that the actual values of φ are quite close to the percolation threshold, i.e., large clusters must be taken into account together with small clusters. The determination of the probabilistic characteristics of clusters, describing the topological structure of the system of lenses, is in this case a very difficult problem, which is best solved by the methods of scaling theory [10].

Here we shall study only some very simple estimates for large clusters, which can be obtained by analogy to the statistical theory of polymer molecules with free rotation [11, 12]. Let a cluster consist of m + 1 overlapping lenses. Consider a broken line whose segments connect successively the centers of the nearest lenses; this line is a two-dimensional analog of a polymer chain. Unlike the polymer chain, however, the length of separate segments is different and cannot be regarded as a determinate quantity — its distribution can be determined from the formulas (2) and (6). This substantially complicates the statistics of such broken lines. For this reason, as a simplification, here we shall neglect both facts, making the assumption that the length of all segments simply equals the quantity $\langle r \rangle$ from (3); the analogy to two-dimensional polymer chains is then complete.

The length of the projection of each segment on some axis x in the plane $l_x = l\cos\theta$, where θ is the angle between the segment and the axis; $l = r = 1/2\sqrt{n}$; and

$$\langle l_x \rangle = \frac{l}{2\pi}\int_0^{2\pi}\cos\theta d\theta = 0, \quad \langle l_x^2 \rangle = \frac{l^2}{2\pi}\int_0^{2\pi}\cos^2\theta d\theta = \frac{l^2}{2}. \tag{12}$$

The number of steps in the positive and negative directions obeys, as is well known, the Bernoulli distribution. If the number of segments is large compared with the difference of the steps in the positive and negative directions, then we obtain a Gaussian distribution for the probability density of the random length X of the straight line connecting the centers of the first and last lenses of the cluster (i.e., the vector sum of all segments) [11, 12]

$$w(X;\ m)=\left(\frac{1}{\pi m l^2}\right)^{1/2}\exp\left(-\frac{X^2}{ml^2}\right) \tag{13}$$

(here the relations (12) were employed). Because the horizon under study is on the average isotropic the mutually orthogonal directions x and y in it are completely equivalent. From here follows an expression for the probability density of the length L of the indicated straight line:

$$w(L;\ m)=\frac{2}{ml^2}\exp\left(-\frac{L^2}{ml^2}\right)L. \tag{14}$$

It follows from (13) and (14), in particular, that $\langle X\rangle = 0$, $\langle X^2\rangle = ml^2/2$ and $\langle L\rangle = \sqrt{\pi m}l/2$, $\langle L^2\rangle = ml^2$.

Now let the x axis be oriented along the vector sum of the segments, and let x = 0 correspond to the center of the right-hand lens. The distribution of the coordinates of the centers of the (k + 1)-st lens in the Gaussian approximation is given by

$$w_k(x_k,\ y_k;\ L,\ m)=\frac{m}{\pi k(m-k)l^2}\exp\left\{-\frac{m}{k(m-k)l^2}\left[\left(x_k-\frac{kL}{m}\right)^2+y_k^2\right]\right\}, \tag{15}$$

whence it follows that $\langle x_k\rangle = kL/m$, $\langle y_k\rangle = 0$ and, further,

$$\langle x_k^2\rangle=\left(\frac{k}{m}\right)^2L^2+\frac{m-k}{m}\frac{kl^2}{2},\quad \langle y_k^2\rangle=\frac{m-k}{m}\frac{kl^2}{2}.$$

The maximum value of $\langle r_k\rangle = \langle x_k^2 + y_k^2\rangle$ is achieved for

$$k=k_m=\frac{l^2m^2}{2(l^2m^2-L^2)},\ l_m^2<2L^2.$$

The value of $\langle r_k^2\rangle$, obtained for $k = k_m$, is the mean-square maximum "radius" of a cluster, containing the (m + 1)-st lens and characterized by the quantity L. Averaging additionally $\langle r_k^2\rangle$ with the help of (14) gives

$$\rho_k^2=\int_0^\infty\langle r_k^2\rangle w(L;\ m)\,dL=kl^2,$$

and it is completely natural to regard $\rho_m = \sqrt{m}l$ as the average "radius" of a cluster. Then the probability for the existence of such a cluster (for the fact that it does not intersect any additional lens of radius R along its outer boundary) can be evaluated as the probability that no center of a lens falls within a ring with area $\pi R(2\rho_m + R)$, i.e., based on (1) we have

$$Q_m(R)\sim\exp[-\pi R(2\rho_m+R)n]\sim\exp(-2\pi R\rho_m n). \tag{16}$$

This formula can serve as a rough estimate of the probability of the appearance of large clusters.

All results presented above were obtained in the Gaussian approximation, for the validity of which the inequalities $m \gg 1$ and $L \ll m\ell$ must be satisfied. If the second inequality does not hold, then the calculations become significantly more complicated, which now in particular lead not to (13)-(15), but rather to more complicated distributions, which depend on Langevin function and are analogous, with respect to their meaning, to the distributions in the Kuhn and Grün theory of linear polymers [11]. If the first inequality does not hold, then the analysis becomes even more difficult. In this case a two-dimensional analog of Treloar's theory of polymer chains [11] can be constructed; some calculations for small m are also presented in [13].

Using these results it is in principle also possible to determine other geometric properties of the broken lines connecting the centers of lenses belonging to clusters of different sizes. However in order to take into account the probabilistic character of the lengths of the segments of these broken lines and the radii of the lenses themselves and to determine the associated relative fraction of the productive area within clusters and the probability of hydraulic coupling of the wells and in order to solve other problems of an analogous character the theory of two-dimensional polymer chains must be significantly extended, which is a very difficult problem. The question of the usefulness of such a generalization should obviously be resolved after the proposed model is carefully compared with the actual data and possibly after some new data on the structure of bazhenites are obtained.

In principle an entirely different approach to the modeling of bazhenite reservoirs, using the methods of percolation theory, that is completely unrelated with the introduction of cavities with definite sizes and shapes is also possible. For example, a separate horizon of a collector can be represented as an array, for which there is a definite probability to find some volume of petroleum at the nodes of the array; in addition, the nodes are connected by means of ties which can be functional or not with a fixed probability. In limiting cases the proposed mixed array problem reduces to classical problems of nodes or ties in the theory of percolation [7, 10].

4. If the number of horizons can be determined with the help of independent methods, then nR_0^2 can be determined based on the known percentage of unproductive wells from the formulas of the type (5). It is also of interest to obtain information about the distribution of lenses over sizes and properties, for which it is natural to use the results of hydraulic tests of wells.

The random character of the parameters characterizing different experimental curves for separate wells represents the total result of the simultaneous action of diverse random factors. They include, first of all, the random permeability and thickness of the cavities in the region of the well bottom, the total number of horizons drained, the shape and size of the clusters formed by the cavities, and the position of the well bottom relative to the boundaries of the clusters. By postulating some correlation couplings between some of these factors and assuming that the statistical properties of the distributions are known for each of them it is in principle possible to determine a distribution of the type presented in [14]. To do so it is also necessary to solve the problem of the flow into the wells, regeneration of the pressure in their vicinity, etc. in regions with a complex geometry. Obviously the problems arising here are very complicated. They are even more complicated due to the lack of complete information about each of the indicated factors, since in order to separate their contributions it is now necessary to solve inverse, rather than direct, problems. For this reason it is best to start from some simplified ideas regarding the structure of the horizons of the type employed in [3].

To this end we shall assume that the thickness of all cavities is the same, but that the cavities are distributed according to their permeability, and we shall study a well whose bottom lies at the center of a circular lens, placed in a fictitious filtering medium, whose permeability is identical to the effective permeability of the horizon as a whole; we shall assume that the random permeability of the lens material is uniform. On the basis of this simple model, in a certain sense generalizing the model of [3], the spread in the experimental curves of the regeneration and drop of the pressure is associated with the random character of the radius and permeability of the lenses.

The effective permeability κ of a medium consisting of freely overlapping circular lenses with arbitrarily distributed radii but identical permeability κ_1 in a matrix with a low permeability κ_0 can be calculated based on the formula [15]

$$\frac{\varkappa}{\varkappa_0}=\frac{1}{2}\left\{(1-2\varphi)\left(1-\frac{\varkappa_1}{\varkappa_0}\right)+\left[(1-2\varphi)^2\left(1-\frac{\varkappa_1}{\varkappa_0}\right)^2+4\frac{\varkappa_1}{\varkappa_0}\right]^{1/2}\right\}. \tag{17}$$

In the case $\kappa_1 \gg \kappa_0$ we obtain from (17) the approximate formulas

$$\varkappa \approx (1-2\varphi)^{-1}\varkappa_0,\ \varphi<0{,}5;$$
$$\varkappa \approx (1-2\varphi)\varkappa_1,\ \varphi>0{,}5, \tag{18}$$

from which, in particular, follows the value $\varphi = 0.5$ as the threshold of percolation (for $\kappa_0 = 0$ the horizon as a whole has a nonzero permeability only in the case $\varphi > 0.5$). If the permeability of the matrix and the lenses are random quantities, then the linearity of the relations (18) permits employing them in this case also with κ_0 and κ_1 replaced by the corresponding average values. Thus in what follows we assume that κ is known. On the basis of this model the random character of the effect of the surrounding medium on the well is taken into account only through the distribution of the radii of the corresponding lenses.

As an example we shall study here only the problem of starting up a well with a constant output. We shall introduce the dimensionless time t and radial coordinate r so that the radius and coefficient of piezoelectric conductivity of the lens would equal unity, and we shall determine the dimensionless pressures inside p_1 and outside p_0 the lens so that in the limit $r \to 0$ the quantity $r(\partial p_1/\partial r)$ equals unity. Then we have the problem

$$\partial p_0/\partial t = \varepsilon\Delta p_0,\ r\geqslant 1;\ \partial p_1/\partial t = \Delta p_1,\ 0\leqslant r<1;$$
$$\lim_{r\to 0} r\frac{\partial p_1}{\partial r} = 1;\ p_0 = p_1,\ \varepsilon\frac{\partial p_0}{\partial r} = \frac{\partial p_1}{\partial r},\ r=1;\ p_i = 0,\ t=0. \tag{19}$$

Here ε is the dimensionless effective piezoelectric conductivity of the medium surrounding the lens, which is formally determined from (18) as $\varepsilon = \kappa$ for $\kappa_1 = 1$. The solution of this problem by the Laplace transform method as well as the asymptotic behavior at short and long times were examined in [16]. Here, to obtain an easily understood result, we shall confine our attention to an approximate solution, following from the zeroth approximation of the method of integral relations [16]. After the calculations we obtain

$$p_1(t,\ r) = \begin{cases} \left.\begin{array}{l} 0,\ r\geqslant 2\sqrt{t} \\ \ln(r/2\sqrt{t}),\ r<2\sqrt{t} \end{array}\right| t\leqslant \dfrac{1}{4}, \\ \ln r - \varepsilon^{-1}\ln\sqrt{1+4\varepsilon\tau},\ \tau = t-1/4>0; \end{cases}$$
$$p_0(t,\ r) = \begin{cases} 0,\ t\leqslant 1/4, \\ \left.\begin{array}{l} 0,\ r\geqslant\sqrt{1+4\varepsilon\tau} \\ \varepsilon^{-1}\ln(r/\sqrt{1+4\varepsilon\tau}),\ r<\sqrt{1+4\varepsilon\tau} \end{array}\right| \tau = t-1/4>0. \end{cases} \tag{20}$$

The formulas (20) are illustrated in Fig. 3 for different values of ε. After the pressure perturbation wave reaches the boundary of the lens the rates of pressure drop at the well bottom become higher than the rates for an unbounded formation, and the increase is all the greater the smaller the ratio of the piezoelectric conductivities of the surrounding medium and the lens itself ε. This result remains true in the case when, instead of introducing a fictitious circumlens medium, clusters with arbitrary configuration are studied. If such a model were applicable, the solution (20) could be used for interpreting test results from separate wells, i.e., actually for solving the inverse problem. We emphasize that the effective radii of the lenses, introduced in this section, are a measure of the linear scales of the regions in which unhindered filtration occurs, and differ from the radii studied in the preceding sections.

In reality, the rates at which the bottom pressure drops decrease with time, as if new draining regions were suddently connected to the well [2]. This effect cannot in principle be explained by factors that prevent filtration or that limit its region. To explain it, it is necessary to assume, analogously to [2, 3], that new regions are added to the filtration process when critical pressure drops or pressure gradients are reached in some zone, or fracture processes near the well bottom, which increase the permeability [1, 5], must be taken into account in an explicit form.

It is obvious based on the foregoing discussion that in order to construct an adequate model of the structure of and filtrational processes in bazhenite reservoirs and to estimate their petroleum reserves diverse and quite fundamental problems, many of which are nontraditional for petroleum-industrial mechanics, must be solved. Some of these problems can be formulated in a manner following directly from the foregoing discussions.

NOTATION

f, probability density distribution; G, P, Q, W, and w, probabilities and probability densities of different events; i, number of horizons; L and X, linear scales of a cluster; l, distance between the centers of neighboring lenses; m, number of lenses in a cluster; n, number density of the centers of lenses in a plane; p, pressure; R, radius of a lens; r, radial coordinate; s, area of a lens; t, time; γ, parameter in (8); ε, dimensionless piezoelectric conductivity of the fictitious circumlens medium; κ, permeability; ρ_m, average radius of a cluster; φ, relative fraction of the area occupied by lenses in a single horizon; $1 - \varphi_*$, relative fraction of the unproductive area in all horizons; and < > denotes averaging.

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